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THE EMISSION OF AN ELEMENTARY SLOT VIBRATOR, LOCATED IN THE CEN--ETC(U)

JAN 79 Y V PIMENOV, L G BRAUDE

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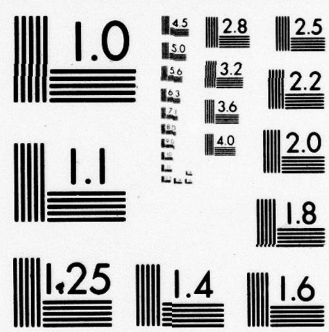

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FOREIGN TECHNOLOGY DIVISION

THE EMISSION OF AN ELEMENTARY SLOT VIBRATOR,
LOCATED IN THE CENTER OF AN IDEALLY CONDUCTING DISK

by

Yu. V. Pimenov, L. G. Braude

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EDITED TRANSLATION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

The Emission of an Elementary Slot Vibrator, Located in the Center of an Ideally Conducting Disk

Yu. V. Pimenov, L. G. Braude

Asymptotic expressions for a field, arising in a remote zone upon the excitation of an ideally conducting disk by an elementary slot vibrator (magnetic dipole), located in the center of the disk, were obtained on the basis of the solution of a strict integral equation.

Introduction

The emission of an elementary slot vibrator, located in the center of an infinitely thin, ideally conducting round disk, was investigated by M. G. Belkina in work [1]. The solution was obtained on the basis of the Fourier method in the form of a series of spheroidal functions. As is known, such series in the case of a large disk, in comparison with the wavelength, converge very slowly, and the solution becomes practically unsuitable for numerical calculations. Thus, of interest is the obtaining of the asymptotic solution for the case $\kappa a \gg 1$, where

$\kappa = \frac{2\pi}{\lambda}$ is the wave number; λ is the wavelength; a is the radius of the disk.

A unilateral slot, cut in a disk, is equivalent to an elementary magnetic vibrator, lying on a disk. In accordance with the principle of duality [2] it is possible, instead of a problem concerning the excitation of a disk by an elementary magnetic vibrator, located in the center of the disk, to solve a problem concerning the excitation of an ideally conducting surface with a round aperture by an elementary electrical vibrator, located in the center of the aperture, and then, in accordance with the known transfer equations, to find the solution of the initial problem. When $\kappa a \gg 1$ the second (supplementary) problem is solved quite simply.

Statement of the Problem

Let us examine a supplementary problem concerning the excitation of an ideally conducting surface with a round aperture of radius a by an elementary electrical vibrator with moment $\vec{p}$, located in the center of the aperture.

Let us introduce a Cartesian coordinate system x, y, z, whose origin coincides with the center of the aperture, axis z is perpendicular to the plane of the screen, and the direction of the x axis coincides with the direction of the moment

of the vibrator $(\vec{p} = \vec{x}_0 p)$. Let us also introduce the cylindrical coordinate system r, φ, z, the z axis of which coincides with the z axis of the Cartesian system (Fig. 1).

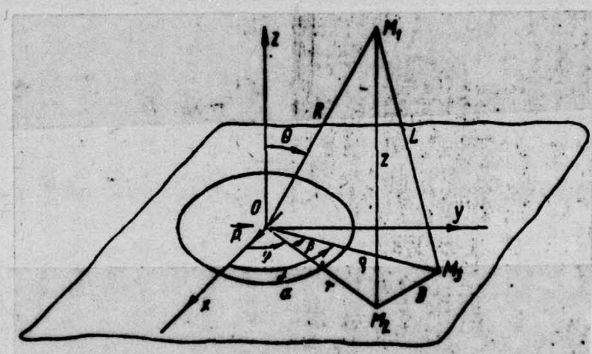

Fig. 1

The strength of the primary electrical field, created by the elementary electric vibrator, is

$$\vec{E}_1^\circ = \vec{r}_0 E_{1r}^\circ + \vec{\varphi}_0 E_{1\varphi}^\circ + \vec{z}_0 E_{1z}, \tag{1}$$

where

$$E_{1r}^\circ = f_1(r, z)\cos\varphi;\; E_{1\varphi}^\circ = f_2(r, z)\sin\varphi;\; E_{1z} = f_3(r, z)\cos\varphi;$$

$$f_1(r, z) = M\frac{e^{-i\kappa R}}{R^3}\left[z^2 - \frac{i}{\kappa R}\left(1 - \frac{i}{\kappa R}\right)(z^2 - 2r^2)\right];$$

$$f_2(r, z) = -M\frac{e^{-i\kappa R}}{R}\left[1 - \frac{i}{\kappa R} - \frac{1}{(\kappa R)^2}\right];$$

$$f_3(r, z) = -M\frac{e^{-i\kappa R}}{R}\frac{r}{R}\left[1 - \frac{3i}{\kappa R} - \frac{3}{(\kappa R)^2}\right]\frac{z}{R};$$

$$M = \frac{p\kappa^2}{4\pi\varepsilon};\; R = \sqrt{r^2 + z^2},$$

and ε is the dielectric constant of the medium. The time dependence is taken in the form $e^{i\omega t}$.

Under the effect of field (1) on the surface with the round aperture currents are induced with a density of

$$\vec{j}(r,\varphi)=\vec{r}_0 j_r(r,\varphi)+\vec{\varphi}_0 j_\varphi(r,\varphi)=\vec{x}_0 j_x(r,\varphi)+\vec{y}_0 j_y(r,\varphi). \tag{2}$$

The vector potential, which corresponds to these is

$$\vec{A}=\frac{\mu}{4\pi}\int_a^\infty \rho\,d\rho\int_\varphi^{2\pi+\varphi}\frac{e^{-i\kappa L}}{L}\,j(\rho,\alpha)\,d\alpha, \tag{3}$$

where

$$L=\sqrt{r^2+\rho^2+z^2-2r\rho\cos(\alpha-\varphi)},$$

and μ is the magnetic permeability of the medium.

G. A. Grinberg showed (see [3] or [4]), that in the case of ideally conducting, infinitely thin shields, the vector potential $\vec{A}$ at points of the shield can be found independent of function $\vec{j}(r,\varphi)$. This makes it possible, applying relationship (3) to points of the shield, to reduce the problem to the solution of an integral equation of the first kind. For determining function $\vec{A}$ on the shield, i.e., when $r>a,\ z=0$, we will proceed in the following manner.

The strength of the secondary electrical field $\vec{E}_1$ is connected with vector potential $\vec{A}$ by relationship

$$\vec{E}_1 = -\operatorname{grad}\Psi - i\omega\vec{A}, \tag{4}$$

where

$$\Psi = \frac{1}{\omega\varepsilon\mu}\operatorname{div}\vec{A}.$$

The following boundary conditions should be achieved on the surface of the shield

$$E_{1r} = -E^{\circ}_{1r} \quad \text{when} \quad r \geqslant a,\ z = 0; \tag{5}$$

$$E_{1\varphi} = -E^{\circ}_{1\varphi} \quad \text{when} \quad r \geqslant a,\ z = 0, \tag{6}$$

which, taling (4) into account, can be rewritten in the form:

$$\frac{\partial\Psi}{\partial r} + i\omega A_r = E^{\circ}_{1r} \quad \text{when} \quad r \geqslant a,\ z = 0; \tag{7}$$

$$\frac{1}{r}\frac{\partial\Psi}{\partial\varphi} + i\omega A_\varphi = E^{\circ}_{1\varphi} \quad \text{when} \quad r \geqslant a,\ z = 0, \tag{8}$$

where A_r and A_φ are respectively the radial and the azimuthal components of vector $\vec{A}$.

Since $E^{\circ}_{1r} = f_1(r, z)\cos\varphi$, and $E^{\circ}_{1\varphi} = f_2(r, z)\sin\varphi$, then in accordance with the results of work [3], the scalar potential Ψ on the surface of

shield can be represented in the following form

$$\Psi = \psi(r)\cos\varphi \quad \text{when} \quad r \geqslant a,\ z = 0, \tag{9}$$

and function $\psi(r)$ should satisfy the condition of emission and differential equation

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{d\psi}{dr} + \left(\kappa^2 - \frac{1}{r^2}\right)\psi = -\left.\frac{\partial f_0}{dz}\right|_{z=0};\ r \geqslant a. \tag{10}$$

Solving (10) by the method of variation of the arbitrary constants and taking into account the condition of emission, we obtain

$$\psi(r) = BH_1^{(2)}(\kappa r) + \frac{\pi i}{4}\left\{H_1^{(1)}(\kappa r)\int_r^\infty F(t)\,H_1^{(2)}(\kappa t)\,dt - \right.$$
$$\left. - H_1^{(2)}(\kappa r)\int_r^\infty F(t)\,H_1^{(1)}(\kappa t)\,dt\right\},\ r \geqslant a, \tag{11}$$

where

$$F(t) = -M\frac{e^{-i\kappa t}}{t^2}\left[1 - \frac{3i}{\kappa t} - \frac{3}{(\kappa t)^2}\right],$$

$H_1^{(1)}$ and $H_1^{(2)}$ are Hankel functionsof the first order of the first and second kind, respectively, and B is a certain constant, which must be determined subsequently from the condition of the vanishing of the radial component of current density at the edge of the aperture:

$$j_r(a) = 0. \tag{12}$$

Thus, function $\psi(r)$, and consequently, function $\Psi(r, z)$ also, when $r \geqslant a$;

z = 0 are determined correct to constant B.

Expressing from (7) and (8) components A_r and A_φ and then changing to the Cartesian components of vector $\vec{A}$, we obtain

$$\vec{A} = \vec{x}_0 A_x + \vec{y}_0 A_y, \tag{13}$$

where

$$\left.\begin{aligned} A_x &= A_x^{(0)}(r) + A_x^{(2)}(r)\cos 2\varphi \text{ при } \overset{\text{when}}{} r \geqslant a;\ z = 0 \\ A_y &= A_y^{(2)}(r)\sin 2\varphi \text{ при } \overset{\text{when}}{} r \geqslant a;\ z = 0 \end{aligned}\right\}; \tag{14}$$

$$\left.\begin{aligned} A_x^{(0)}(r) &= \frac{i}{2\omega}\left[\frac{d\psi}{dr} - f_1(r, 0) + \frac{1}{r}\psi + f_2(r, 0)\right] \\ A_x^{(2)}(r) = A_y^{(2)}(r) &= \frac{i}{2\omega}\left[\frac{d\psi}{dr} - f_1(r, 0) - \frac{1}{r}\psi - f_2(r, 0)\right] \end{aligned}\right\}. \tag{15}$$

Applying (3) to points of the shield $(r \geqslant a,\ z = 0)$ and taking (13) into account, we arrive at two independent integral equations of the first kind:

$$A_x^{(0)}(r, 0) + A_x^{(2)}(r, 0)\cos 2\varphi = \frac{\mu}{4\pi}\int_a^\infty \rho\, d\rho \int_\varphi^{\varphi+2\pi} \frac{e^{-i\varkappa D}}{D} j_x(\rho, \alpha)\, d\alpha; \tag{16}$$

$$A_y^{(2)}(r, 0)\sin 2\varphi = \frac{\mu}{4\pi}\int_a^\infty \rho\, d\rho \int_\varphi^{\varphi+2\pi} \frac{e^{-i\varkappa D}}{D} j_y(\rho, \alpha)\, d\alpha, \tag{17}$$

where

$$D = \sqrt{r^2 + \rho^2 - 2r\rho\cos(\alpha - \varphi)}.$$

From the form of the left sides of equations (16), (17), it follows, that

functions $j_x(\rho, \alpha)$ and $j_y(\rho, \alpha)$ can be sought in the form

$$\left.\begin{aligned} j_x &= j_x^{(0)}(\rho) + j_x^{(2)}(\rho)\cos 2\alpha \\ j_y &= j_y^{(2)}(\rho)\sin 2\alpha \end{aligned}\right\}, \tag{18}$$

and $j_y^{(2)}(\rho = j_x^2\,\rho)$.

Substituting (18) in (16) and proceeding in the internal integral to a new variable of integration β with respect to formula $\beta = \alpha - \varphi$, we arrive at two independent integral equations of the first kind for the functions $j_x^{(0)}(\rho)$ and $j_x^{(2)}(\rho)$:

$$A_x^{(\nu)}(r, 0) = \frac{\mu}{4\pi}\int_a^\infty j_x^{(\nu)}(\rho)\,\rho\,d\rho\int_0^{2\pi}\frac{e^{-i\kappa D}}{D}\cos\nu\beta\,d\beta;\ r \geqslant 0;\ \nu = 0;\ 2. \tag{19}$$

where

$$D = \sqrt{r^2 + \rho^2 - 2r\rho\cos\beta}.$$

The left sides of equations (19) are known correct to constant B, which should be determined after finding functions $j_x^{(0)}(\rho)$ and $j_x^{(2)}(\rho)$. For calculating constant B there serves condition (12), which after the transition to functions $j_x^{(0)}(\rho)$ and $j_x^{(2)}(\rho)$ takes the form

$$j_x^{(0)}(a) + j_x^{(2)}(a) = 0. \tag{20}$$

Thus, the problem concerning the excitation of an ideally conducting surface with a round aperture by an elementary electrical vibrator, located in the center of the aperture, is reduced to the solution of two independent integral equations of the first kind (19) with the additional condition (20).

Determining the Currents

Equations (19) are strict integral equations of the problem. They are valid with any values of parameter ka. We are interested in the solution of these equations when $\kappa a \gg 1$. In this case the left sides of equations (19), determined by formulas (15), are substantially simplified. Since $\kappa a \gg 1$, and $r \geqslant a$, then it is possible to replace the Hankel functions going into the left sides of equations (19) by the first terms of their asymptotic expansions:

$$H_\nu^{(2)}(\kappa r) \approx \frac{\sqrt{2}}{\sqrt{\pi \kappa r}} e^{-i\kappa r} e^{i\left(\frac{\pi}{4} - \frac{\nu\pi}{2}\right)}. \tag{21}$$

Disregarding, moreover, terms of the order $\frac{1}{\kappa r}$ in comparison with unity, we obtain

$$\omega p \left[B' \frac{e^{-i\kappa r}}{\sqrt{r}} - \delta_\nu i \frac{e^{-i\kappa r}}{\sqrt{r}} \right] = \int_a^\infty j_x^{(\nu)}(\rho) \rho \, d\rho \int_0^{2\pi} \frac{e^{-i\kappa D}}{D} \cos \nu\beta \, d\beta; \quad r \geqslant a; \quad \nu = 0;\ 2, \tag{22}$$

where

when

$$B' = B \frac{2\sqrt{2\pi}\, e^{i\frac{3\pi}{4}}}{p\kappa\sqrt{\kappa}} \varepsilon; \quad \delta_\nu = \begin{cases} 1 & \text{при } \nu = 0, \\ 0 & \text{при } \nu = 2. \end{cases}$$

when

It is possible to transform the internal integral on the right side of (22), by employing the asymptotic equality, proved in work [5]:

$$\int_0^{2\pi} \frac{e^{-i\kappa D}}{D} \cos\nu\beta\, d\beta = -\frac{\pi i}{\sqrt{r\rho}} \{H_0^{(2)}(\kappa|r-\rho|) + i(-1)^\nu H_0^{(2)}[\kappa(r+\rho)]\} + O[(\kappa a)^{-3/2}],\quad \nu = 0;\ 2. \tag{23}$$

Substituting (23) in (22) and introducing dimensionless variables ξ, η and γ, connected with ρ, r and κ to relationships

$$\rho = a(1+\xi),\quad r = a(1+\eta),\quad \gamma = \kappa a, \tag{24}$$

we will obtain

$$\int_0^\infty u^{(\nu)}(\xi) H_0^{(2)}(\gamma|\eta-\xi|)\,d\xi + i\int_0^\infty u^{(\nu)}(\xi) H_0^{(2)}[\gamma(\eta+\xi+2)]\,d\xi = = Ce^{i\frac{\pi}{4}} \frac{\sqrt{2}}{\sqrt{\pi\gamma}} e^{-i\gamma\eta} - \delta_\nu H_0^{(2)}[\gamma(\eta+1)], \tag{25}$$

where

$$u^{(\nu)}(\xi) = \frac{\sqrt{2\pi}\, e^{-i\frac{\pi}{4}a^2}}{i\omega\sqrt{\gamma p}} j_x^{(\nu)}[a(1+\xi)]\sqrt{1+\xi},\quad \nu = 0;\ 2, \tag{26}$$

and $C = -i\sqrt{a}\, e^{-i\gamma}B'$ is a certain constant, which will be subsequently determined from condition (20).

Employing the equalities, proved by G. A. Grinberg

$$\int_0^\infty \frac{e^{-i\gamma(\xi+R_1)}\sqrt{R_1}}{\pi\sqrt{\xi(\xi+R_1)}} H_0^{(2)}(\gamma|\eta-\xi|)\,d\xi = H_0^{(2)}[\gamma(\eta+R_1)]. \tag{27}$$

$$\int_0^\infty \frac{\sqrt{\gamma}\, e^{-i\frac{\pi}{4}}}{\sqrt{2\pi\zeta}} e^{-i\gamma\xi} H_0^{(2)}(\gamma|\eta-\xi|)\,d\xi = e^{-i\gamma\eta}, \tag{28}$$

we transform the equations of (25) into integral equations of the second kind:

$$u^{(\nu)}(\xi) = -i\frac{e^{-i\gamma(\xi+2)}}{\pi\sqrt{\xi}}\int_0^\infty \frac{u^{(\nu)}(t)\,e^{-i\gamma t}\sqrt{t+2}}{\xi+t+2}\,dt + \\ + C\frac{e^{-i\gamma\xi}}{\pi\sqrt{\xi}} - \delta_\nu \frac{e^{-i\gamma(\xi+1)}}{\pi\sqrt{\xi}(\xi+1)}, \quad \nu = 0;\ 2. \tag{29}$$

Since by assumption $\gamma = \kappa a \gg 1$, then the solution of the equations of (29) can be found by the method of successive approximations. However, it is more convenient to employ an artificial method.

The functions of $u^{(\nu)}(\xi)$ are proportional to the components $j^{(\nu)}[a(1+\xi)]$ of the current density, induced in the shield. With an increase in variable ξ functions $u^{(\nu)}(\xi)$ decrease in absolute value and vanish when $\xi\to\infty$. Thus, when $\gamma\gg 1$ the main contribution to the value of the integral, going into (29), gives the neighborhood of point $\xi = 0$. Consequently, the following approximate equality occurs

$$u^{(\nu)}(\xi) = -i\frac{\sqrt{2}}{\pi}\frac{e^{-i\gamma(\xi+2)}}{\sqrt{\xi}(\xi+2)}U_0^{(\nu)} + C\frac{e^{-i\gamma\xi}}{\pi\sqrt{\xi}} - \\ -\delta_\nu \frac{e^{-i\gamma(\xi+1)}}{\pi(\xi+1)}, \quad \nu = 0;\ 2, \tag{30}$$

where

$$U_0^{(\nu)} = \int_0^\infty u^{(\nu)}(\xi)\, e^{-i\sqrt{\gamma}\xi}\, d\xi, \quad \nu = 0;\ 2. \tag{31}$$

It is possible to strictly show, that the error in equality (30) does not exceed $O(\gamma^{-3/2})$.

For determining constants $U_0^{(\nu)}$ let us multiply both sides of (30) by $e^{-i\sqrt{\gamma}\xi}$ and let us integrate ξ from zero to infinity. As a result we will arrive at two (for v = 0 and v = 2) independent algebraic equations, by solving which, we will obtain

$$U_0^{(\nu)} = \frac{\dfrac{C e^{i\frac{\pi}{4}}}{\sqrt{2}\sqrt{\pi\gamma}} - \delta_\nu\left[1 - \Phi(\sqrt{i2\gamma})\right]}{1 + i e^{i2\gamma}\left[1 - \Phi(\sqrt{i4\gamma})\right]}, \quad \nu = 0;\ 2, \tag{32}$$

where

$$\Phi(\sqrt{iw}) = \frac{2e^{i\frac{\pi}{4}}}{\sqrt{\pi}} \int_0^{\sqrt{w}} e^{-is^2}\, ds. \tag{33}$$

It remains to determine constant C. Employing the condition of (20), which after the transition to functions $u^{(\nu)}(\xi)$, takes the form

$$u^{(0)}(0) + u^{(2)}(0) = 0, \tag{34}$$

we obtain

$$C = \frac{e^{-i\gamma}}{2}\, \frac{1 + i e^{i2\gamma}\left[1 - \Phi(\sqrt{i4\gamma})\right] - \dfrac{i}{\sqrt{2}}\left[1 - \Phi(\sqrt{i2\gamma})\right]}{1 + i e^{i2\gamma}\left[1 - \Phi(\sqrt{i4\gamma})\right] - i\,\dfrac{e^{-i2\gamma}\, e^{-i\pi/4}}{2\sqrt{\pi\gamma}}}. \tag{35}$$

Expression (35) is considerably simplified, if we replace function $\Phi\left(\sqrt{i\bar{w}}\right)$ by its asymptotic representation. In this case, the following simple relationship $C=\frac{1}{2}e^{-i\gamma}+O\left(\gamma^{-3/2}\right)$ will be fulfilled.

Thus, the functions $u^{(\nu)}(\xi)$ are completely determined, consequently, the distribution of currents, induced on the shield, is known.

Determining the Field

Let us proceed to the determination of the field, arising upon the excitation of an ideally conducting surface with a round aperture by an elementary electrical vibrator, place in the center of the aperture.

The vector potential of the currents, induced on the shield, is expressed by formula (3) and has two components: A_x and A_y, in which

$$\left.\begin{aligned} A_x &= A_x^{(0)}(r,\ z)+A_x^{(2)}(r,\ z)\cos 2\varphi \\ A_y &= A_y^{(2)}(r,\ z)\sin 2\varphi;\ A_y^{(2)}=A_x^{(2)} \end{aligned}\right\}. \qquad (36)$$

On the shield $(r\geqslant a,\ z=0)$ of function $A_{(x}^{\nu)}(r,\ z)$ coincide with the earlier introduced functions $A_x^{(\nu)}(r)$, and at an arbitrary point in space are determined by the following expression

$$A_x^{(\nu)}=\frac{\mu}{4\pi}\int_a^\infty j_x^{(\nu)}(\rho)\,\rho\,d\rho\int_0^{2\pi}\frac{e^{-i\varkappa L}}{L}\cos\nu\beta\,d\beta,\ \nu=0;\ 2. \qquad (37)$$

Formula (37) is not convenient for numerical calculations. Let us find its asymptotic representation. In this case, we will distinguish two regions: the first - adjacent to the z axis, and the second - the remaining part of space.

Let us first examine the second region. Let us introduce a spherical system of coordinates R, θ, φ , the polar axis of which coincides with the z axis of the cylindrical system of coordinates (Fig. 1).

Formula (37) in this coordinate system takes the form

$$A_x^{(\nu)} = \frac{i e^{i\frac{\pi}{4}} \omega\mu_a \sqrt{\gamma}}{4\pi\sqrt{2\pi a}} \int_0^\infty a^{(\nu)}(\xi)\sqrt{1+\xi}\,d\xi \int_0^{2\pi} \frac{e^{-i\gamma_0 L_0}}{L_0} \times \cos\nu\beta\,d\beta, \quad \nu = 0;\ 2, \tag{38}$$

where

$$L_0 = L/a = [\,r_0^2 + (1+\xi)^2 - 2r_0(1+\xi)\sin\theta\cos\beta]^{1/2}, \quad r_0 = R/a.$$

Since in the examined region inequality $\gamma\sin\theta \gg 1$, is fulfilled, then the internal integral in (38) can be trnasformed in accordance with the formula (see work [7])

$$\int_0^{2\pi} \frac{e^{-i\gamma L_0}}{L_0}\cos\nu\beta\,d\beta = -\frac{\pi i}{\sqrt{r_0(1+\xi)\sin\theta}}\,[H_0^{(2)}(\gamma b) + i(-1)^\nu H_0^{(2)}(\gamma d)] + O[(\gamma\sin\theta)^{-3/2}], \quad \nu = 0;\ 2, \tag{39}$$

where

$$b = [\, r_0^2 + (1+\xi)^2 - 2r_0(1+\xi)\sin\theta]^{1/2},$$
$$d = [\, r_0^2 + (1+\xi)^2 + 2r_0(1+\xi)\sin\theta]^{1/2}.$$

Substituting in (38) the values of functions $u^{(v)}(\xi)$ from (30) and employing formula (39), we obtain

$$A_x^{(v)} = -\frac{i\, e^{i\frac{\pi}{4}}\omega\mu p\sqrt{\gamma}}{4\sqrt{2\pi\sin\vartheta\, a}\sqrt{r_0}}\{U_0^{(v)}[Q_1(2,\ r_0,\ \theta) + $$
$$+ iQ(2,\ r_0,\ \theta+\pi)] + iC[Q_2(r_0,\ \theta) + iQ_2(r_0,\ \theta+\pi)] -$$
$$- i\delta_v[Q_1(1,\ r_0\theta) + iQ_1(1,\ r_0,\ \theta+\pi]\}, \tag{40}$$

where

$$Q_1(\sigma,\ r_0,\ \theta) = \frac{1}{\pi}\int_0^\infty \frac{e^{-i\gamma(\xi+\sigma)}\sqrt{\sigma}}{\sqrt{\xi}\,(\xi+\sigma)} H_0^{(2)}(\gamma b)\, d\xi; \tag{41}$$

$$Q_2(\sigma, r_0,\ \theta) = \frac{1}{\pi}\int_0^\infty \frac{e^{-i\gamma\xi}}{\sqrt{\xi}} H_0^{(2)}(\gamma b)\, d\xi. \tag{42}$$

Integral $Q_1(\sigma,\ r_0,\ \theta)$ was examined in detail in work [7]. The following asymptotic equality is valid in the far zone (when $r_0 \to \infty$):

$$Q_1(\sigma,\ r_0,\ \theta) = \frac{\sqrt{2}\, e^{-i\frac{\pi}{4}}}{\sqrt{\pi\gamma}}\ \frac{e^{-i\gamma r_0}}{\sqrt{r_0}}\, e^{-i\gamma(\sigma-1)\sin\theta}\{1 -$$
$$- \Phi(\sqrt{i\gamma\sigma(1-\sin\theta)})\} + O(r_0^{-3/2}). \tag{43}$$

Integral $Q_2(r_0,\ \theta)$ was calculated in work [8] and was equal to:

$$Q_2(r_0,\ \theta) = \frac{\sqrt{2}}{\sqrt{\pi\gamma}}\ e^{i\frac{\pi}{4}} e^{-i\gamma(r_0-1)\sin\theta}[1 - \Phi(\sqrt{i\gamma r_0(1-\sin\theta)})]. \tag{44}$$

In the far zone (when $r_0 \to \infty$) expression (44) takes the form

$$Q_2(r_0, \theta) = \frac{\sqrt{2}}{\pi\gamma} \frac{e^{-i\gamma r_0}}{\sqrt{r_0}} \frac{e^{i\gamma \sin\theta}}{\sqrt{1-\sin\theta}} + O(r_0^{-3/2}). \quad (45)$$

Employing relationships (43) and (45) and changing from components $A_x^{(0)}$ and $A_x^{(2)}$ in a Cartesian coordinate system to components A_φ and A_θ in a spherical coordinate system, we obtain:

$$A_\varphi = \frac{\omega\mu}{4\pi} \frac{p}{a} \frac{e^{-i\gamma r_0}}{r_0} S_\varphi(\theta) \sin\varphi; \quad (46)$$

$$A_\theta = \frac{\omega\mu}{4\pi} \frac{p}{a} \frac{e^{-i\gamma r_0}}{r_0} S_\theta(\theta) \cos\varphi, \quad (47)$$

where

$$S_\varphi(\theta) = \frac{1}{\sqrt{\sin\theta}} \{[U_0^{(2)} - U_0^{(0)}] F_1(\theta) + F_2(\theta)\};$$

$$S_\theta(\theta) = \frac{\cos\theta}{\sqrt{\sin\theta}} \Bigg\{ [U_0^{(2)} + U_0^{(0)}] F_1(\theta) - F_2(\theta) + \quad (48)$$

$$+ C \frac{2e^{-i\frac{\pi}{4}}}{\sqrt{\pi\gamma}} \left(i \frac{e^{i\gamma\sin\theta}}{\sqrt{1-\sin\theta}} - \frac{e^{-i\gamma\sin\theta}}{\sqrt{1+\sin\theta}} \right) \Bigg\};$$

$$F_1(\theta) = e^{-i\gamma\sin\theta} [1 - \Phi(\sqrt{i2\gamma(1-\sin\theta)})] +$$

$$+ i e^{i\gamma\sin\theta} [1 - \Phi(\sqrt{i2\gamma(1+\sin\theta)})]; \quad (49)$$

$$F_2(\theta) = i[1 - \Phi(\sqrt{i\gamma(1-\sin\theta)})] - [1 - \Phi(\sqrt{i\gamma(1+\sin\theta)})].$$

The strength of the secondary electrical field $\vec{E}_1$ in the far zone is connected with vector potential $\vec{A}$ by relationship $\vec{E}_1 = -i\omega\vec{A}$. Consequently, the vector components of the strength of the complete electrical field $\vec{E} = \vec{E}_1 + \vec{E}_1^0$ in the far zone in the region $\gamma\sin\theta \gg 1$ are equal, respectively, to:

$$E_\varphi = -\frac{e^{-i\gamma r_0}}{r_0}\,\frac{i\gamma^2 p \sin\varphi}{4\pi a^3 \varepsilon}\,[S_\varphi(\theta) - i]; \tag{50}$$

$$E_\theta = -\frac{e^{-i\gamma r_0}}{r_0}\,\frac{i\gamma^2 p \cos\varphi}{4\pi a^3 \varepsilon}\,[S_\theta(\theta) + i]. \tag{51}$$

Let us proceed to the calculation of the field in the region, adjacent to the z axis, and let us limit ourselves to an examination of the far zone.

Assuming in (38) $L_0 \approx r_0 - (1+\xi)\sin\theta\cos\beta$ and changing the order of integration, we obtain

$$A_x^{(\nu)} = \frac{i\,e^{i\frac{\pi}{4}}\,\omega\mu\, p\sqrt{\gamma}}{4\pi\sqrt{2\pi}\,a}\,\frac{e^{-i\gamma r_0}}{r_0}\int_0^{2\pi} G^{(\nu)}(\beta)\,e^{i\gamma\sin\theta\cos\beta}\times \times \cos\nu\beta\, d\beta,\quad \nu = 0;\ 2, \tag{52}$$

where

$$G^{(\nu)}(\beta) = \int_0^\infty u^{(\nu)}(\xi)\sqrt{1+\xi}\,e^{i\gamma\xi\sin\theta\cos\beta}\,d\xi,\quad \nu = 0;\ 2. \tag{53}$$

Integral (53) can be calculated asymptotically. Substituting in (53) the values of functions $u^{(\nu)}(\xi)$ from (30) and disregarding terms of the order $O\{[\gamma(1-\sin\theta)]^{-3/2}\}$, we obtain

$$G^{(\nu)}(\beta) = \frac{1}{\sqrt{2\pi\gamma}}\,\frac{e^{-i\frac{\pi}{4}}}{\sqrt{1-\sin\theta\cos\beta}}\,K^{(\nu)} + O\{[\gamma(1-\sin\theta)]^{-3/2}\}, \tag{54}$$

where

$$\left.\begin{aligned} K^{(0)} &= -i\,U_0^{(0)}\,e^{-i2\gamma} + \sqrt{2}\,C - \sqrt{2}\,e^{-i\gamma} \\ K^{(2)} &= -i\,U_0^{(2)}\,e^{-i2\gamma} + \sqrt{2}\,C \end{aligned}\right\}. \tag{55}$$

Expanding $(1-\sin\theta\cos\beta)^{-1/2}$ in a series with respect to powers $\sin\theta\cos\beta$ and being limited to the first three terms of the expansion, we arrive after term-by-term integration in formula (52) to the following expression:

$$A_x^{(\nu)} = \frac{i\omega\mu p}{4\pi a}\frac{e^{-i\gamma r_0}}{r_0}K^{(\nu)}T^{(\nu)}(0), \quad \nu = 0;\ 2, \tag{56}$$

where

$$T^{(0)}(\theta) = J_0(\gamma\sin\theta) + \frac{1}{2}\sin\theta J_1(\gamma\sin\theta) + \\ + \frac{3}{16}\sin^2(\theta)[J_0(\gamma\sin\theta) - J_2(\gamma\sin\theta)]; \tag{57}$$

$$T^{(2)}(\theta) = -J_2(\gamma\sin\theta) + \frac{1}{4}\sin\theta[J_1(\gamma\sin\theta) - \\ - J_3(\gamma\sin\theta)] + \frac{3}{32}\sin^2\theta[J_0(\gamma\sin\theta) - 2J_2(\gamma\sin\theta) + J_4(\gamma\sin\theta)]. \tag{58}$$

Here J_n is a Bessel function of the first kind of order n. Proceeding to components A_φ and A_θ in the spherical coordinate system, we obtain:

$$A_\varphi = \frac{i\omega\mu p}{4\pi a}\frac{e^{-i\gamma r_0}}{r_0}V_\varphi(\theta)\sin\varphi; \tag{59}$$

$$A_\theta = \frac{i\omega\mu p}{4\pi a}\frac{e^{-i\gamma r_0}}{r_0}V_\theta(\theta)\cos\varphi, \tag{60}$$

where

$$V_\varphi(\theta) = K^{(2)}T^{(2)}(\theta) - K^{(0)}T^{(0)}(\theta); \tag{61}$$

$$V_\theta(\theta) = [K^{(2)}T^{(2)}(\theta) + K^{(0)}T^{(0)}(\theta)]\cos\theta. \tag{62}$$

Consequently, the components of the vector of the strength of the complete electrical field in the far zone in region $\gamma(1-\sin\theta) \gg 1$ are equal to:

$$E_{\varphi} = \frac{e^{-i\gamma r_0}}{r_0}\frac{\gamma^2 p \sin\varphi}{4\pi a^3 \varepsilon}[V_{\varphi}(\theta) - 1]; \quad (63)$$

$$E_{\theta} = \frac{e^{-i\gamma r_0}}{r_0}\frac{\gamma^2 p \cos\varphi}{4\pi a^3 \varepsilon}[V_{\theta}(\theta) + 1]. \quad (64)$$

Thus, the additional problem concerning the excitation of an ideally conducting surface with a round aperture by an elementary electrical vibrator, located in the center ofthe aperture, is completely solved. Let us proceed to the analysis of the original problem.

Excitation of a Disk by an Elementary Magnetic Vibrator

The electromagnetic field, created by an elementary magnetic vibrator (a unilateral slot), located in the center of an ideally conducting disk, can be found with the aid of the principle of duality [2], employing the obtained solution. In the case, in the far zone in region $\gamma \sin\theta \gg 1$ the strength of the complete electrical field $\vec{E}'$ is determined with the following expressions:

a) in the upper half-space $(z>0)$:

$$E'_{\theta} = H'_{\varphi}\sqrt{\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0}\frac{i\gamma^2 m \sin\varphi}{4\pi a^3 \varepsilon}[S_{\varphi}(\theta) - 2i], \quad (65)$$

$$E'_{\varphi} = H'_{\theta}\sqrt{\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0}\frac{i\gamma^2 m \sin\varphi}{4\pi a^3 \varepsilon}[S_{\varphi}(\theta) + 2i], \quad (66)$$

where m is the moment of the vibrator;

b) in the lower half-space $(z<0)$:

$$E'_\theta = H'_\varphi \sqrt{\frac{\mu}{\varepsilon}} = \frac{e^{-i\gamma r_0}}{r_0} \frac{i\gamma^2 m \sin\varphi}{4\pi a^3 \varepsilon} S_\varphi(0), \tag{67}$$

$$E'_\varphi = H'_\theta \sqrt{\frac{\mu}{\varepsilon}} = \frac{e^{-i\gamma r_0}}{r_0} \frac{i\gamma^2 m \cos\varphi}{4\pi a^3 \varepsilon} S_\theta(\theta). \tag{68}$$

Accordingly, in the region, adjacent to the z axis [i.e., with the fulfillment of inequality $\gamma(1-\sin\theta) \gg 1$], the field in the far zone is determined with formulas:

a) in the upper half-space:

$$E'_\theta = H'_\varphi \sqrt{\frac{\mu}{\varepsilon}} = \frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \sin\varphi}{4\pi a^3 \varepsilon} [V_\varphi(\theta) - 2], \tag{69}$$

$$E'_\varphi = H'_\theta \sqrt{\frac{\mu}{\varepsilon}} = \frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \cos\varphi}{4\pi a^3 \varepsilon} [V_\theta(\theta) + 2]; \tag{70}$$

b) in the lower half-space:

$$E'_\theta = H'_\varphi \sqrt{\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \sin\varphi}{4\pi a^3 \varepsilon} V_\varphi(\theta), \tag{71}$$

$$E'_\varphi = H'_\theta \sqrt{\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \cos\varphi}{4\pi a^3 \varepsilon} V_\theta(\theta). \tag{72}$$

Numerical Results

Numerical calculations were carried out for the case $\gamma=5$ for comparing the obtained asymptotic expressions (65)-(72) with the results of strict solution [1].

Fig. 2 shows a standardized diagram of the directivity of the elementary magnetic vibrator, located in the center of an ideally conducting disk on the

upper side of the disk, corresponding to plane $\varphi=90^\circ$. The solid line shows the values of the E_θ' component, relative to the maximum value of modulus $|E_\theta'|$, taken from work [1] (strict solution). The broken line shows the plot of analogous values, calculated in accordance with formulas (65), (67), (69), (71).

Fig. 3 shows a standardized diagram of the directivity in the plane $\varphi=0^\circ$. The solid line corresponds to the strict solution, the broken line to the values, calculated in accordance with formulas (66), (68), (70) and (72).

Fig. 4 shows the standardized diagram of the directivity of the elementary magnetic vibrator, located in the center of an ideally conducting disk, in the plane $\varphi=90^\circ$, calculated in accordance with formulas (65), (67), (69), (71) when $\gamma=10$.

Fig. 5 shows the standardized diagram of the directivity in plane $\varphi=0^\circ$, calculated in accordance with formulas (66), (68), (70), (72) when $\gamma=10$.

Fig. 6 and 7 show the plots of analogous diagrams when $\gamma = 15$.

As the calculations show, formulas (65)-(72) span the entire range in the variation of angle θ.

The obtained solution will be the more accurate, the greater is the magnitude of $\gamma = \kappa a$. However, as the numerical calculations show, it satisfactorily depicts the character of the directivity diagrams even at such a small value of γ, as $\gamma = 5$.

The obtained solution is valid only when the magnetic vibrator lies on the disk, however, the methodology employed in theis work makes it possible to obtain a solution also for the case of a magnetic vibrator, raised above the disk.

In conclusion, the authors wish to thank Prof. G. Z. Ayzenberg and Docent L. S. Korol'kevich for their discussion of the present work and for their valuable advice.

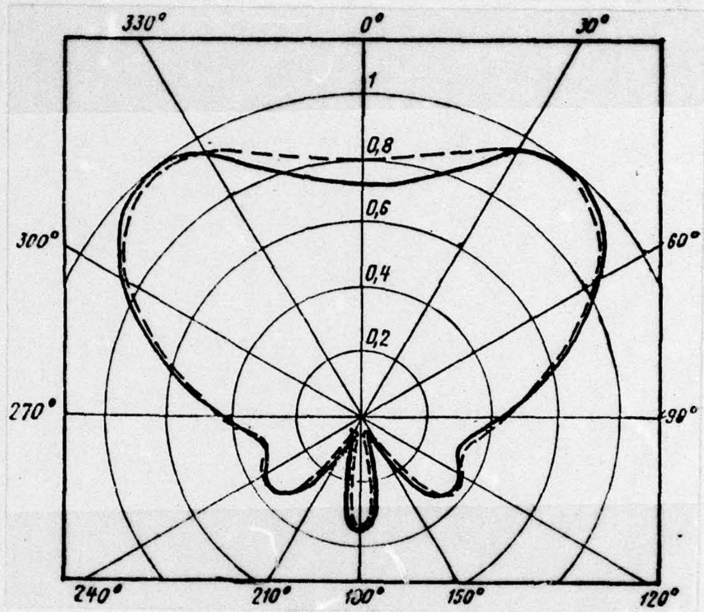
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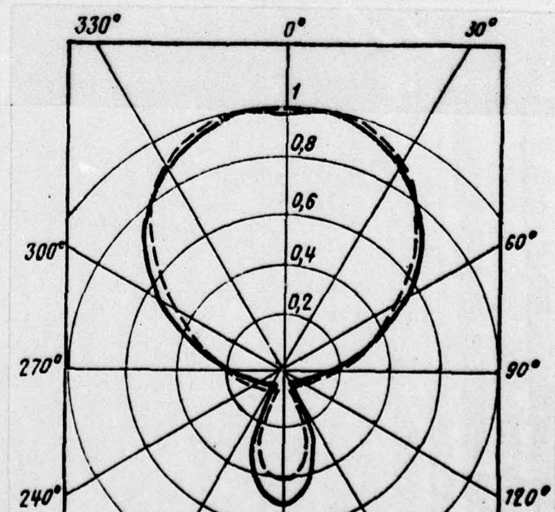
Fig. 2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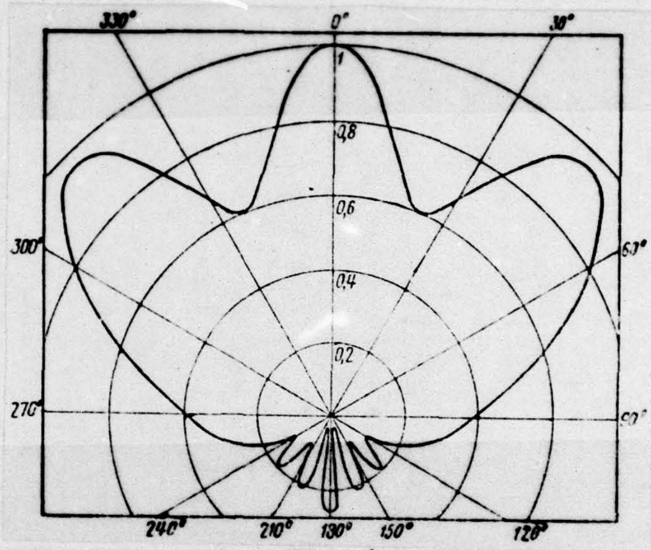

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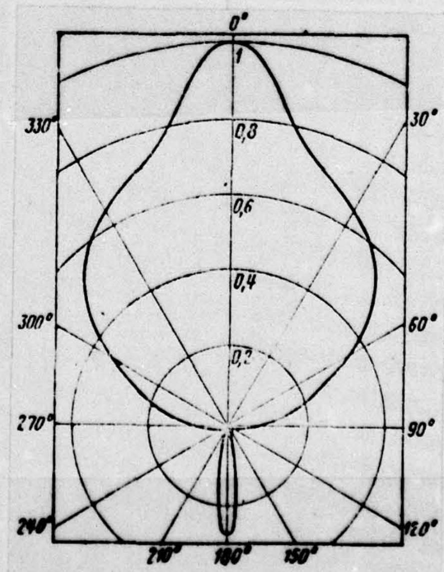

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Fig. 5

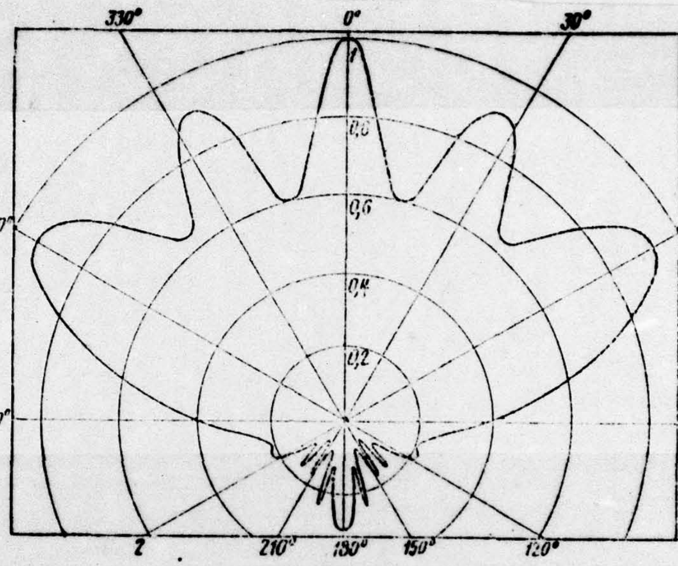

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Fig. 6

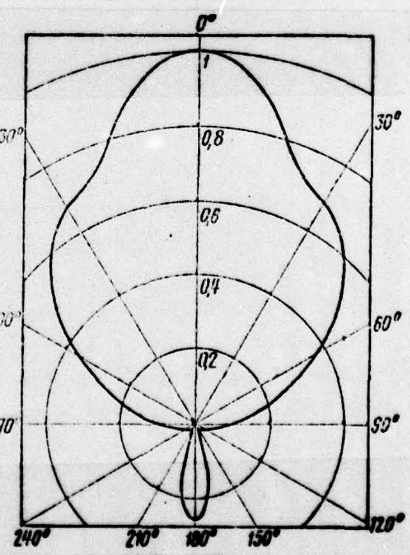

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Fig. 7

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